

GCE

Further Mathematics A

Y534/01: Discrete Mathematics

Advanced Subsidiary GCE

Mark Scheme for Autumn 2021

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

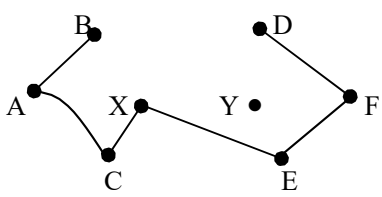
© OCR 2021

Annotations and abbreviations

Annotation in RM assessor	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

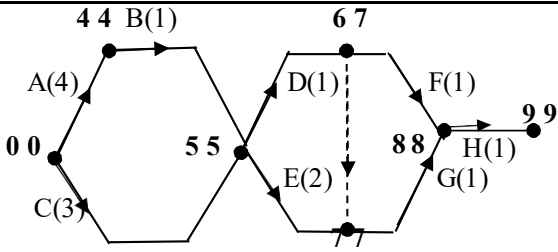
Question		Answer	Marks	AO	Guidance
1	(a)	5 partitions into a set of size 1 and a set of size 4 $\{X X, X, X, X\}$	B1	1.1	5 where smaller set has size 1 or ${}^5C_1 = 5$
		10 partitions into a set of size 2 and a set of size 3 $\{X, X X, X, X\}$ because there are ${}^5C_2$ choices for the set of size 2	B1	2.5	10 where smaller set has size 2, with an explanation of why it is 10 (note the total of 15 is given in the question) e.g. ${}^5C_2 = 10$ or $(5 \times 4) \div 2 = 10$ or $4 + 3 + 2 + 1 = 10$
		Alternative solution $\{A\}, \{B, C, D, E\}$ $\{B\}, \{A, C, D, E\}$ $\{C\}, \{A, B, D, E\}$ $\{D\}, \{A, B, C, E\}$ $\{E\}, \{A, B, C, D\}$	B1		List (or any equivalent) that has exactly 5 distinct cases where smaller set has size 1 May just list one set, e.g. $\{A\}, \{B\}, \{C\}, \{D\}, \{E\}$
		$\{A, B\}, \{C, D, E\}$ $\{A, C\}, \{B, D, E\}$ $\{A, D\}, \{B, C, E\}$ $\{A, E\}, \{B, C, D\}$ $\{B, C\}, \{A, D, E\}$ $\{B, D\}, \{A, C, E\}$ $\{B, E\}, \{A, C, D\}$ $\{C, D\}, \{A, B, E\}$ $\{C, E\}, \{A, B, D\}$ $\{D, E\}, \{A, B, C\}$	B1		List (or any equivalent) that has 10 distinct cases sets where smaller set has size 2 May just list one set, e.g. $\{A, B\}, \{A, C\}, \{A, D\}, \{A, E\}, \{B, C\}, \{B, D\}, \{B, E\}, \{C, D\}, \{C, E\}, \{D, E\}$
			[2]		
1	(b)	Partitions into sets of sizes 1, 1 and 3 $5 \times 4 \div 2 = 10$ partitions of this type	M1	1.1	Considering cases where set sizes are 1, 1, 3
			A1	2.1	Explanation of why there are 10 of these e.g. ${}^5C_3 = 10$ or $5 \times 4 \div 2 = 10$ or a list of the cases
		Partitions into sets of sizes 1, 2 and 2 $5 \times ({}^4C_2 \div 2) = 5 \times 3 = 15$ partitions of this type	M1	1.1	Considering cases where set sizes are 1, 2, 2
			A1	2.1	Explaining why there are 15 of these e.g. a relevant calculation or list of cases
			[4]		
1	(c)	10 partitions into sets of sizes 1, 1, 1, 2	M1	2.1	Trying to deal with the cases when there are more than 3 subsets
		1 partition into sets of sizes 1, 1, 1, 1, 1			May be implied from answer 51
		$15 + 25 + 10 + 1 = 51$	A1	1.1	51
			[2]		
1	(d)	Number line is split into 6 pieces	B1	2.1	6 pieces
		But there are 8 numbers	B1	2.2a	Using pigeonhole, or explaining why there must be at least one piece with two or more numbers
		Hence result by the pigeonhole principle			
			[2]		

Question			Answer	Marks	AO	Guidance
2	(a)	(i)	Next-fit method			
			Bin 1 12	M1	1.1	Bins 1 and 2 correct
			Bin 2 23			
			Bin 3 15	A1	1.1	All correct
			Bin 4 18 8			
2	(a)	(ii)	Bin 5 7 5	[2]		
			First-fit method			
			Bin 1 12 15	M1	1.1	Bins 1 and 2 correct
			Bin 2 23 7			
			Bin 3 18 8	A1	1.1	All correct
2	(a)	(iii)	Bin 4 5			
			Bin 5	[2]		
			23 18 15 12 8 7 5			Ordered list may be seen
			First-fit decreasing method			
			Bin 1 23 7	M1	1.1	Bins 1 and 2 correct
2	(b)		Bin 2 18 12			
			Bin 3 15 8 5	A1	1.1	All correct
			Bin 4			
			Bin 5	[2]		
2	(b)		With 'online' lists the items are presented one at a time and the whole list is not known until the end.	B1	1.2	Evidence of understanding what 'online' means
			With next-fit and first-fit the items are placed in the order they appear in the list, so these methods can be used 'offline' or 'online'.	B1	2.3	Evidence of realising that ffd cannot be used with an online list (or implied from an appropriate statement about next-fit and first-fit)
			However, for first-fit decreasing the whole list needs to be known before it can be sorted, so first-fit decreasing can only be used for an 'offline' list.	[2]		

Question			Answer	Marks	AO	Guidance																																										
2	(c)		$88 \div 4 = 22$, so M is at least 22 But it is not possible to fill 4 bins of capacity 22 Since $22 - 18 = 4$ which is less than 5 So the 23 would have to be split as 4 and 19 And then there is no 3 to go with the 19 $M = 23$ is possible e.g. $23 - x$ and x , $18 + 5$, $15 + 8$, $12 + 7$ Hence, least M is 23	M1 A1 B1 [3]	1.1 2.4 2.2a	Identifying that M must be at least 22 Showing that $M = 22$ is not possible Fully correct explanation Showing that $M = 23$ is possible																																										
3	(a)		ABXE	B1 [1]	1.1																																											
3	(b)		<table border="1"> <tr><td>A</td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td>0.6</td><td>B</td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td>1.1</td><td>1.7</td><td>C</td><td></td><td></td><td></td><td></td></tr> <tr><td>2.7</td><td>2.1</td><td>2.5</td><td>D</td><td></td><td></td><td></td></tr> <tr><td>2.8</td><td>2.2</td><td>1.8</td><td>1.2</td><td>E</td><td></td><td></td></tr> <tr><td>3.3</td><td>2.7</td><td>2.5</td><td>0.6</td><td>0.7</td><td>F</td><td></td></tr> </table>	A							0.6	B						1.1	1.7	C					2.7	2.1	2.5	D				2.8	2.2	1.8	1.2	E			3.3	2.7	2.5	0.6	0.7	F		M1 A1 M1 A1 [4]	1.1 1.1 1.1 1.1	$AB = 0.6$, $AC = 1.1$ $AD = 2.7$, $AE = 2.8$ $DF = 0.6$, $EF = 0.7$ $BF = 2.7$, $CF = 2.5$ $AF = 3.3$ or ft from other values
A																																																
0.6	B																																															
1.1	1.7	C																																														
2.7	2.1	2.5	D																																													
2.8	2.2	1.8	1.2	E																																												
3.3	2.7	2.5	0.6	0.7	F																																											
3	(c)		$AB = 0.6$ $AC = 1.1$ $CE = 1.8$ $EF = 0.7$ $DF = \underline{0.6}$ 4.8 	M1 A1 B1 ft [3]	3.1b 3.2 a 1.1	A graph that connects $\{A, B, C, D, E, F\}$ with or without X and/or Y Correct tree drawn or arcs listed, including CX and XE 4.8 (km) or total for their tree																																										
3	(d)		Adapting the answer to part (c) $B - A - C - X - Y - E - F - D$	M1 A1	3.1b 1.1	Any walk or cycle that starts at B and uses every vertex at least once, including X and Y cao																																										
				[2]																																												

Question			Answer	Marks	AO	Guidance																												
4	(a)	(i)	<table><tr><td colspan="2">Points won by Mia</td><td>X</td><td>Mia Y</td><td>Z</td></tr><tr><td>X</td><td>4</td><td>15</td><td>9</td></tr><tr><td>Y</td><td>11</td><td>6</td><td>5</td></tr><tr><td>Z</td><td>10</td><td>5</td><td>1</td></tr></table>	Points won by Mia		X	Mia Y	Z	X	4	15	9	Y	11	6	5	Z	10	5	1	B1 [1]	1.1	15, 9 and 6 all correct											
			Points won by Mia		X	Mia Y	Z																											
			X	4	15	9																												
			Y	11	6	5																												
Z	10	5	1																															
4	(a)	(ii)	<table><tr><td colspan="2">Points won by Li</td><td>X</td><td>Mia Y</td><td>Z</td></tr><tr><td>X</td><td>0.5</td><td>-10.5</td><td>-4.5</td></tr><tr><td>Y</td><td>-6.5</td><td>-1.5</td><td>-0.5</td></tr><tr><td>Z</td><td>-5.5</td><td>-0.5</td><td>3.5</td></tr></table>	Points won by Li		X	Mia Y	Z	X	0.5	-10.5	-4.5	Y	-6.5	-1.5	-0.5	Z	-5.5	-0.5	3.5	B1 [1]	1.1	Subtract 4.5 from each value in the table showing points won by Li All correct Or any positive multiple of this table											
			Points won by Li		X	Mia Y	Z																											
			X	0.5	-10.5	-4.5																												
			Y	-6.5	-1.5	-0.5																												
			Z	-5.5	-0.5	3.5																												
4	(b)		<table><tr><td colspan="2">Points won by Li</td><td>X</td><td>Mia Y</td></tr><tr><td>X</td><td>0.5</td><td>-10.5</td></tr><tr><td>Z</td><td>-5.5</td><td>-0.5</td></tr></table>	Points won by Li		X	Mia Y	X	0.5	-10.5	Z	-5.5	-0.5	B1 B1 [2]	1.1 1.1	Row Y removed, seen in resulting 2×2 table Column Z removed, seen in resulting 2×2 table Follow through their table from part (a)(ii) or using original table <table><tr><td>e.g.</td><td>X</td><td>Y</td></tr><tr><td>X</td><td>1</td><td>-21</td></tr><tr><td>Z</td><td>-11</td><td>-1</td></tr></table> <table><tr><td>e.g.</td><td>X</td><td>Y</td></tr><tr><td>X</td><td>5</td><td>-6</td></tr><tr><td>Z</td><td>-1</td><td>4</td></tr></table>	e.g.	X	Y	X	1	-21	Z	-11	-1	e.g.	X	Y	X	5	-6	Z	-1	4
			Points won by Li		X	Mia Y																												
			X	0.5	-10.5																													
			Z	-5.5	-0.5																													
			e.g.	X	Y																													
			X	1	-21																													
Z	-11	-1																																
e.g.	X	Y																																
X	5	-6																																
Z	-1	4																																
4	(c)		Row minima are −10.5 and −5.5 Row maximin = −5.5 Li's play-safe is strategy Z Mia should play strategy X	M1 A1 ft B1 ft [3]	1.1 1.1 1.1	Follow through their table from part (b) or original tables Working may be seen on table from part (b) Seen or implied from row minima, or identifying play-safe for Li Or indicated in table, or implied from their row maximin value stated Correct choice for their table from part (b)																												

Question			Answer	Marks	AO	Guidance															
5	(a)	(i)		M1 M1 M1 A1 [4]	1.1 1.1 1.1 1.1	Ignore any extra lines (e.g. profit lines) or working for parts (b), (c) Line $2x + 3y = 12$ through (6, 0) and (0, 4) Line $x + y = 10$ through at least two of (10, 0), (2, 8), (4, 6), (6, 4), (8, 2) and (0, 10) Line $5x + 2y = 30$ through at least two of (6, 0), (4, 5), (2, 10) and (0, 15) Feasible region identified and correct															
5	(a)	(ii)	<table><tr><td>x</td><td>y</td><td>$P = 4x - y$</td></tr><tr><td>0</td><td>4</td><td>-4</td></tr><tr><td>0</td><td>10</td><td>-10</td></tr><tr><td>3.33</td><td>6.67</td><td>6.67</td></tr><tr><td>6</td><td>0</td><td>24</td></tr></table> Maximum $P = 24$	x	y	$P = 4x - y$	0	4	-4	0	10	-10	3.33	6.67	6.67	6	0	24	M1 A1 [2]	3.1a 1.1	‘Determine’ so method must be seen, not implied Checking at least two of their vertices or sliding a profit line (a line of gradient 4 anywhere on graph or indicating the vertex (6, 0)) 24
x	y	$P = 4x - y$																			
0	4	-4																			
0	10	-10																			
3.33	6.67	6.67																			
6	0	24																			
5	(b)		FR has boundaries $x = 0$, $x + y = k$, $2x + 3y = 12$ $x + y = k$ and $2x + 3y = 12$ or $4x - y = 3$ Profit line $4x - y = 3$ cuts $2x + 3y = 12$ at (1.5, 3) $k = 4.5$ Alternative solution $4x - (k - x) = 3 \Rightarrow 3x - k = 3$ and $2x + 3(k - x) = 12 \Rightarrow 3k - x = 12$ $k = 4.5$	M1 M1 A1 M1 M1 A1 [3]	3.4 3.1a 2.2a	Not graphical Vertex where $2x + 3y = 12$ and $x + y = k$ or profit on line $x + y = k$ Calculate where profit = 3 on boundary $2x + 3y = 12$ or (1.5, 3) 4.5 Use $x + y = k$ to substitute for y (or x) in $4x - y = 3$ Form a second simultaneous equation in the same unknowns 4.5															

Question			Answer	Marks	AO	Guidance
5	(c)		Profit line $4x - y = 3$ cuts $5x + 2y = 30$ at $\frac{3}{13}, \frac{1}{13}$ $k = \frac{141}{13}$	M1 A1 [2]	3.1a 2.2a	Not graphical Or $2\frac{1}{13}, 8\frac{1}{13}$ or (2.7 to 2.8, 8.0 to 8.2) Or $10\frac{11}{13}$ or 10.8 to 10.9
6	(a)	(i)	 Minimum time = 9 hours A, B, E, G, H have no float	M1 A1 M1 M1 A1 A1 [6]	3.3 1.1 3.4 3.4 1.1 1.1	Activity network with A, B and C correct D, E, F, G, H and dummy correct (accept directions missing) Forward pass attempted, or implied from min duration correct Backward pass attempted, or implied from critical activities correct 9 A, B, E, G, H (in any order) and no others
6	(a)	(ii)	Assuming that there are enough workers for each activity Resourcing may restrict <u>how many</u> activities can happen together	B1 [1]	3.5b	A reason why it may not always be possible to do all the activities that are needed at the same time NOT an assumption about the durations or immediate predecessors or that would delay the start time of an activity (e.g. weather or delays in arrival of materials)
6	(b)		Earliest time that E can start is 5 hours from start If there are not enough workers then A, B, C may need to be done one after another, taking 8 hours. And E could also be delayed until after D and F, giving a latest start time for E of 10 hours	B1 M1 A1 [3]	1.1 3.5a 2.2b	5 (all the activities that <i>must</i> be done before E have min completion time 5) Recognising that tasks may be done sequentially (or implied from answer 8, 9 or 10) 10 (all the activities that <i>can</i> be done before E have total duration 10 – starting E after 10 would be an unnecessary delay)
6	(c)		Extend the duration of D to 3 hours	B1 [1]	3.5c	Or add an activity immediately after D of duration 2 hours

OCR (Oxford Cambridge and RSA Examinations)
The Triangle Building
Shaftesbury Road
Cambridge
CB2 8EA

OCR Customer Contact Centre

Education and Learning

Telephone: 01223 553998

Facsimile: 01223 552627

Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored